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CP-Violating Profile of the Electroweak Bubble Wall

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Abstract

In any scenario of the electroweak baryogenesis, the profile of the CP violating bubble wall, created at the first-order phase transition, plays an essential role. We attempt to determine it by solving the equations of motion for the scalars in the two-Higgs-doublet model at the transition temperature. According to the parameters in the potential, we found three solutions. Two of them smoothly connect the CP-violating broken phase and the symmetric phase, while the other connects CP-conserving vacua but violates CP in the intermediate region. We also estimate the chiral charge flux, which will be turned into the baryon density in the symmetric phase by the sphaleron process.

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1 Introduction

Since the fascinating proposal of the electroweak baryogenesis[1], its various aspects have been investigated by many authors. For the baryogenesis to occur, the following conditions must be met: (1) The electroweak phase transition (EWPT) must be first order to realize a state out of equilibrium. (2) There had to be CP violation in the era of the EWPT. Besides these, to keep the generated baryons, (3) the sphaleron processes must decouple just after the EWPT. These impose some restrictions on the models of the electroweak theory. The conditions (1) and (3) give an upper bound on the lightest neutral Higgs particle, which is inconsistent with the present lower bound of the Higgs scalar in the minimal standard model. Further to have efficient CP violation, an extension of the Higgs sector would be needed. On this ground, the two-Higgs-doublet model, including the minimal supersymmetric standard model (MSSM), has been studied to estimate the generated baryon number.

It is essential in the scenarios of the baryogenesis, spontaneous or charge transport, to know the profile of the CP violation near the expanding bubble wall created at the EWPT. In the literatures, however, some functional forms of the CP violation were assumed without any reasoning. They must be determined by the dynamics of the scalar fields near the EWPT. One may expect that, in the lowest order of the approximation, spacetime-varying CP violation would be governed by the classical equations of motion of the gauge-Higgs system, in which the Higgs potential is replaced with the effective potential at the transition temperature. This amounts to find the critical bubble, which would be a good approximation to an expanding bubble if the EWPT proceeds calmly.

In this paper, we shall follow this line to obtain the functional form of the CP-violating phase in the two-Higgs-doublet model. In section 2, we derive the equation for the CP-violating phase assuming that the moduli of the Higgs scalars take the kink shape with the same width. In section 3, we enumerate possible boundary conditions of the phase for various choices of parameters in the potential. Next we present some numerical solutions and the chiral charge flux in section 4. The final section is devoted to discussions.

2 The Equation for the Phase

2.1 The Equations of Motion

The system we concern is governed by the lagrangian,

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \sum_{i=1,2}\left(D_\mu\Phi_i\right)^\dagger D^\mu\Phi_i - V_{eff}(\Phi_1,\Phi_2;T), \tag{2.1}$$

where

$$D_\mu \Phi_i(x) \equiv (\partial_\mu - ig\frac{\tau^a}{2}A^a_\mu(x) - i\frac{g'}{2}B_\mu(x))\Phi_i(x).$$

We adopted V_{eff} at T near the transition temperature as the Higgs potential. We are now interested in the classical solution of the bubble-wall shape which mediates between the broken and symmetric phases. If the phase transition proceeds calmly, it will be valid to expect that the bubble wall grows keeping the profile of the critical bubble, which is determined by the static equations of motion. Further, when the bubble is spherically symmetric or is sufficiently macroscopic so that it is regarded as a planar object, the system is reduced to an effective one-dimensional one. In general, in $1+1$-dimensional gauge theories, gauge fields have no dynamical degrees of freedom, that is, they are pure gauge. Here we assume that the gauge fields are written in the pure-gauge form:

$$ig\frac{\tau^a}{2}A^a_\mu(x) = \partial_\mu U_2(x)U_2^{-1}(x), \qquad i\frac{g'}{2}B_\mu(x) = \partial_\mu U_1(x)U_1^{-1}(x),$$

where U_2 and U_1 are elements of $SU(2)_L$ and $U(1)_Y$, respectively. Since we can completely gauge away these gauge fields, we only need to consider the equations of motion for the Higgs fields. Assuming that $U(1)_{em}$ is not broken anywhere, the vacuum expectation values (VEVs) of the Higgs fields have the following form:

$$\langle \Phi_i(x) \rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\rho_i(x)\mathrm{e}^{i\theta_i(x)} \end{pmatrix}, \qquad (i=1,2). \tag{2.2}$$

Now the equations of motion are

$$\begin{aligned} -\partial^2\rho_i(x) + \rho_i(x)\partial_\mu\theta_i(x)\partial^\mu\theta_i(x) - \frac{\partial V_{eff}}{\partial\rho_i} &= 0, \\ \partial_\mu\left(\rho_i^2(x)\partial^\mu\theta_i(x)\right) + \frac{\partial V_{eff}}{\partial\theta_i} &= 0. \end{aligned}$$

Along with these equations, we have the "sourcelessness condition", which arises from the requirement for the gauge fields to be pure-gauge type:

$$\rho_1^2(x)\partial_\mu\theta_1(x) + \rho_2^2(x)\partial_\mu\theta_2(x) = 0.$$

Regarding the bubble wall as a static planar object, these equations are reduced to (taking z as the coordinate perpendicular to the wall)

$$\frac{d^2\rho_i(z)}{dz^2} - \rho_i(z)\left(\frac{d\theta_i(z)}{dz}\right)^2 - \frac{\partial V_{eff}}{\partial\rho_i} = 0, \tag{2.3}$$

$$\frac{d}{dz}\left(\rho_i^2(z)\frac{d\theta_i(z)}{dz}\right) - \frac{\partial V_{eff}}{\partial\theta_i} = 0, \tag{2.4}$$

$$\rho_1^2(z)\frac{d\theta_1(z)}{dz} + \rho_2^2(z)\frac{d\theta_2(z)}{dz} = 0. \tag{2.5}$$

For later convenience, let us change the variable z to y defined by

$$y = \frac{1}{2}\left(1 - \tanh(az)\right), \tag{2.6}$$

where a^{-1} characterizes the width of the wall. In terms of this new coordinate, the above equations are written as

$$\begin{aligned}
4a^2 y(1-y)\frac{d}{dy}\left[y(1-y)\frac{d\rho_i(y)}{dy}\right] - 4a^2y^2(1-y)^2\rho_i(y)\left(\frac{d\theta_i(y)}{dy}\right)^2 - \frac{\partial V_{eff}}{\partial \rho_i} &= 0, && (2.7)\\
4a^2 y(1-y)\frac{d}{dy}\left[y(1-y)\rho_i^2(y)\frac{d\theta_i(y)}{dy}\right] - \frac{\partial V_{eff}}{\partial \theta_i} &= 0, && (2.8)\\
\rho_1^2(y)\frac{d\theta_1(y)}{dy} + \rho_2^2(y)\frac{d\theta_2(y)}{dy} &= 0. && (2.9)
\end{aligned}$$

In order to solve these equations, one must know the explicit form of V_{eff}. Because of the gauge invariance, V_{eff} is a function of $\theta_1 - \theta_2$. From this fact, (2.8) with $i = 2$ is automatically satisfied as long as ρ_i and θ_i satisfy (2.8) with $i = 1$ and (2.9).

2.2 Ansatz for the effective potential

In general, it is difficult to solve the coupled equations (2.7) and (2.8) with the constraint (2.9). When the EWPT proceeds calmly, we expect that the modulus of the Higgs, $\rho_i(z)$, has the shape of a kink, and that ρ_1 and ρ_2 have the same order of width.[1] Assuming that ρ_1 and ρ_2 are the kink type of the same width but with different amplitudes, the problem is now to solve (2.8) in the background of ρ_i. This assumption, in turn, restricts the form of the effective potential. We shall solve the equations for θ_i without specifying any model, but with the potential matching this assumption.

We require that (2.7) has the kink-type solutions in the absence of CP violation ($\theta \equiv \theta_1 - \theta_2 = 0$ or π);

$$\rho_i(y) = v_i(1-y), \tag{2.10}$$

where

$$v_1 = v\cos\beta, \qquad v_2 = v\sin\beta.$$

Then (2.7) is

$$4a^2 v_i y(1-y)(1-2y) + \left.\frac{\partial V_{eff}(\rho_1, \rho_2, \theta = 0 \text{ or } \pi))}{\partial \rho_i}\right|_{\rho_i = v_i(1-y)} = 0. \tag{2.11}$$

For this equation to be satisfied, the form of V_{eff} is somewhat restricted. Now let us determine V_{eff} in terms of a polynomial of ρ_1 and ρ_2.

[1] We implicitly assumed that both the VEVs acquire nonzero values at about the same temperature.

The most general tree-level Higgs potential is given by

$$\begin{aligned} V_0 &= m_1^2\Phi_1^\dagger\Phi_1 + m_2^2\Phi_2^\dagger\Phi_2 + (m_3^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}) \\ &+ \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) - \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &- \left\{\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + [\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)](\Phi_1^\dagger\Phi_2) + \mathrm{h.c.}\right\}, \end{aligned} \tag{2.12}$$

where $m_1^2, m_2^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4 \in \mathbf{R}$ and $m_3^2, \lambda_5, \lambda_6, \lambda_7 \in \mathbf{C}$, three of their phases are independent and yield the explicit CP violation. When all these parameters are real (*i.e.*, no explicit CP violation), we have

$$\begin{aligned} V_0(\rho_1, \rho_2, \theta) &= \frac{1}{2}m_1^2\rho_1^2 + \frac{1}{2}m_2^2\rho_2^2 + m_3^2\rho_1\rho_2\cos\theta + \frac{\lambda_1}{8}\rho_1^4 + \frac{\lambda_2}{8}\rho_2^4 \\ &+ \frac{\lambda_3 - \lambda_4}{4}\rho_1^2\rho_2^2 - \frac{\lambda_5}{4}\rho_1^2\rho_2^2\cos(2\theta) - \frac{\lambda_6}{2}\rho_1^3\rho_2\cos\theta - \frac{\lambda_7}{2}\rho_1\rho_2^3\cos\theta. \end{aligned} \tag{2.13}$$

In the following we shall examine the spontaneously CP-violating and CP-conserving cases. The CP-conserving case is realized if $\theta = 0$ or π. It is sufficient to consider the former, since the latter is obtained by redefining one of the scalars. Without any CP violation, we have, in terms of ρ_i,

$$\begin{aligned} V_0 &= \frac{1}{2}m_1^2\rho_1^2 + \frac{1}{2}m_2^2\rho_2^2 + m_3^2\rho_1\rho_2 + \frac{\lambda_1}{8}\rho_1^4 + \frac{\lambda_2}{8}\rho_2^4 \\ &+ \frac{\tilde\lambda_3}{4}\rho_1^2\rho_2^2 - \frac{1}{2}(\lambda_6\rho_1^2 + \lambda_7\rho_2^2)\rho_1\rho_2, \end{aligned} \tag{2.14}$$

where $\tilde\lambda_3 = \lambda_3 - \lambda_4 - \lambda_5$.

In order to have kink solutions for ρ_i, we need ρ^3-terms with negative coefficients, which are expected to arise at finite temperature. [2] Hence we adopt the following ansatz for the effective potential:

$$V_{eff}(\rho_1, \rho_2, \theta = 0) = V_0(\rho_1, \rho_2, 0) - \left(A\rho_1^3 + B\rho_1^2\rho_2 + C\rho_1\rho_2^2 + D\rho_2^3\right). \tag{2.15}$$

Since we expect this to represent the effective potential with first-order phase transition, the origin $(\rho_1, \rho_2) = (0, 0)$ and the point $(\rho_1, \rho_2) = (v\cos\beta, v\sin\beta)$ must be local minima. This condition amounts to

$$\det\left(\frac{\partial^2 V_{eff}}{\partial\rho_i\partial\rho_j}\right) > 0, \qquad \frac{\partial^2 V_{eff}}{\partial\rho_1^2} > 0 \text{ or } \frac{\partial^2 V_{eff}}{\partial\rho_2^2} > 0, \tag{2.16}$$

at each point. At $(0, 0)$, this reduces to

$$m_1^2 m_2^2 - m_3^4 > 0, \quad \text{and } m_1^2 > 0 \text{ or } m_2^2 > 0. \tag{2.17}$$

[2]Although we do not use the high-temperature expansion, these terms may be considered to represent the effect which causes the first-order phase transition.

Requiring that (2.11) is satisfied with this potential, some of the parameters are expressed by the others. Now we assume $\cos\beta\sin\beta \neq 0$. This condition would be necessary to give masses to up- and down-type quarks when they couple to different Higgs doublets, as in the MSSM. Then the effective potential in the absence of CP violation is

$$\begin{aligned}
V_{eff}(\rho_1,\rho_2,0) &= (2a^2-\frac{1}{2}m_3^2\tan\beta)\rho_1^2+(2a^2-\frac{1}{2}m_3^2\cot\beta)\rho_2^2+m_3^2\rho_1\rho_2 \\
&- \left\{A\rho_1^3+\left[-2A\cot\beta+D\tan^2\beta+\frac{4a^2}{v\sin\beta}(3-\frac{1}{\cos^2\beta})\right]\rho_1^2\rho_2\right. \\
&\quad \left.+\left[A\cot^2\beta-2D\tan\beta+\frac{4a^2}{v\cos\beta}(3-\frac{1}{\sin^2\beta})\right]\rho_1\rho_2^2+D\rho_2^3\right\} \\
&+ \frac{\lambda_1}{8}\rho_1^4+\frac{\lambda_2}{8}\rho_2^4+\frac{\tilde{\lambda}_3}{4}\rho_1^2\rho_2^2 \qquad (2.18)\\
&- \frac{1}{8}\left\{\left[\frac{3}{2}\lambda_1\cot\beta-\frac{\lambda_2}{2}\tan^3\beta+\tilde{\lambda}_3\tan\beta-\frac{8a^2}{v^2\sin\beta\cos\beta}(4-\frac{1}{\cos^2\beta})\right]\rho_1^3\rho_2\right. \\
&\quad \left.+\left[-\frac{\lambda_1}{2}\cot^3\beta+\frac{3}{2}\lambda_2\tan\beta+\tilde{\lambda}_3\cot\beta-\frac{8a^2}{v^2\sin\beta\cos\beta}(4-\frac{1}{\sin^2\beta})\right]\rho_1\rho_2^3\right\},
\end{aligned}$$

and the condition (2.17) is written as

$$4a^2 > \frac{m_3^2}{\sin\beta\cos\beta}. \qquad (2.19)$$

Note that although we use the same notations for the parameters in V_{eff} as those in V_0, their meanings are different. Those in V_{eff} contain radiative as well as finite-temperature corrections near T_C.

Now let us introduce the CP-violating phase into (2.18) in a gauge-invariant manner. Comparing (2.13) and (2.14) suggests that the phase θ is introduced in V_{eff} of (2.18) as

$$\begin{aligned}
\rho_1\rho_2 &\to \rho_1\rho_2\cos\theta, \\
\lambda_5\rho_1^2\rho_2^2 &\to \lambda_5\rho_1^2\rho_2^2\cos(2\theta), \\
\rho_1^3\rho_2 &\to \rho_1^3\rho_2\cos\theta, \\
\rho_1\rho_2^3 &\to \rho_1\rho_2^3\cos\theta.
\end{aligned}$$

On the other hand, we have no principle to determine θ-dependence of ρ^3-terms. Here we investigate the two possibilities:

- no θ-dependence in the ρ^3-terms,
- $\rho_1^2\rho_2 \to \rho_1^2\rho_2\cos\theta$, $\rho_1\rho_2^2 \to \rho_1\rho_2^2\cos\theta$.

Hence our ansatz for the effective potential is

$$\begin{aligned}
& V_{eff}(\rho_1, \rho_2, \theta) \\
= \; & (2a^2 - \frac{1}{2}m_3^2 \tan\beta)\rho_1^2 + (2a^2 - \frac{1}{2}m_3^2 \cot\beta)\rho_2^2 + m_3^2 \rho_1 \rho_2 \cos\theta \\
- \; & \left\{ A\rho_1^3 + \left[-2A\cot\beta + D\tan^2\beta + \frac{4a^2}{v\sin\beta}(3 - \frac{1}{\cos^2\beta}) \right] \rho_1^2 \rho_2 (\cos\theta) \right. \\
& \left. + \left[A\cot^2\beta - 2D\tan\beta + \frac{4a^2}{v\cos\beta}(3 - \frac{1}{\sin^2\beta}) \right] \rho_1 \rho_2^2 (\cos\theta) + D\rho_2^3 \right\} \\
+ \; & \frac{\lambda_1}{8}\rho_1^4 + \frac{\lambda_2}{8}\rho_2^4 + \frac{\lambda_3 - \lambda_4}{4}\rho_1^2\rho_2^2 - \frac{\lambda_5}{4}\rho_1^2\rho_2^2 \cos(2\theta) \qquad (2.20) \\
- \; & \frac{1}{8} \left\{ \left[\frac{3}{2}\lambda_1 \cot\beta - \frac{\lambda_2}{2}\tan^3\beta + \tilde{\lambda}_3 \tan\beta - \frac{8a^2}{v^2 \sin\beta\cos\beta}(4 - \frac{1}{\cos^2\beta}) \right] \rho_1^3 \rho_2 \right. \\
& \left. + \left[-\frac{\lambda_1}{2}\cot^3\beta + \frac{3}{2}\lambda_2 \tan\beta + \tilde{\lambda}_3 \cot\beta - \frac{8a^2}{v^2 \sin\beta\cos\beta}(4 - \frac{1}{\sin^2\beta}) \right] \rho_1 \rho_2^3 \right\} \cos\theta.
\end{aligned}$$

Here $\cos\theta$ in the ρ^3-terms is unity in the case of the first possibility.

2.3 Equations for the phases

Once θ-dependence of V_{eff} is determined, one can derive the equations for θ_i from (2.8). In our case, the sourcelessness condition (2.9) is reduced to

$$\cos^2\beta \frac{d\theta_1}{dy} + \sin^2\beta \frac{d\theta_2}{dy} = 0. \qquad (2.21)$$

As noted above, we only need to solve the equation for θ_1, as long as θ_1 and θ_2 satisfy the sourcelessness condition. Since, from this condition, $\theta_2(y)$ is written as

$$\theta_2(y) = -\theta_1(y)\cot^2\beta + \text{const.},$$

we have

$$\theta(y) = \frac{1}{\sin^2\beta}(\theta_1(y) + \text{const.}).$$

Noting that the derivative terms in the equation for θ_1 are invariant under the shift of θ_1, the constant in the r.h.s. of the above equation can be ignored. Putting

$$\theta(y) = \frac{1}{\sin^2\beta}\theta_1(y), \qquad (2.22)$$

we have

$$y^2(1-y)^2 \frac{d^2\theta(y)}{dy^2} + y(1-y)(1-4y)\frac{d\theta(y)}{dy}$$

$$
\begin{aligned}
&= \frac{1}{4a^2\sin^2\beta\cos^2\beta}\left[-m_3^2\sin\beta\cos\beta - \left(A\cos^3\beta + D\sin^3\beta - \frac{4a^2}{v}\right)v(1-y)\right. \\
&\qquad \left. + \frac{v^2}{8}(\lambda_1\cos^4\beta + \lambda_2\sin^4\beta + 2\tilde{\lambda}_3\sin^2\beta\cos^2\beta - \frac{16a^2}{v^2})(1-y)^2\right]\sin\theta(y) \\
&\qquad + \frac{\lambda_5 v^2}{4a^2}(1-y)^2\sin\theta(y)\cos\theta(y). \qquad (2.23)
\end{aligned}
$$

For later convenience, we denote the coefficients in the above equation as

$$
\begin{aligned}
b &\equiv -\frac{m_3^2}{4a^2\sin\beta\cos\beta}, \\
c &\equiv \frac{v^2}{32a^2}(\lambda_1\cot^2\beta + \lambda_2\tan^2\beta + 2\tilde{\lambda}_3) - \frac{1}{2\sin^2\beta\cos^2\beta} \\
&= \frac{v^2}{8a^2}(\lambda_6\cot\beta + \lambda_7\tan\beta), \qquad (2.24) \\
d &\equiv \frac{\lambda_5 v^2}{4a^2}. \\
e &\equiv \frac{v}{4a^2\sin^2\beta\cos^2\beta}\left(A\cos^3\beta + D\sin^3\beta - \frac{4a^2}{v}\right) \\
&= -\frac{v}{4a^2}\left(\frac{B}{\sin\beta} + \frac{C}{\cos\beta}\right).
\end{aligned}
$$

If there is no θ-dependence in the ρ^3-terms in the V_{eff}, $e = 0$. Because of the condition (2.19),

$$
b > -1. \qquad (2.25)
$$

On the other hand, at $(\rho_1, \rho_2) = (v\cos\beta, v\sin\beta)$

$$
\begin{aligned}
\frac{\partial^2 V_{eff}}{\partial\rho_1^2} &= m_1^2 - (8a^2 e - 12a^2 c + \tilde{\lambda}_3 v^2)\sin^2\beta, \\
\frac{\partial^2 V_{eff}}{\partial\rho_2^2} &= m_2^2 - (8a^2 e - 12a^2 c + \tilde{\lambda}_3 v^2)\cos^2\beta, \\
\frac{\partial^2 V_{eff}}{\partial\rho_1\partial\rho_2} &= m_3^2 + (8a^2 e - 12a^2 c + \tilde{\lambda}_3 v^2)\sin\beta\cos\beta,
\end{aligned}
$$

so that

$$
\det\left(\frac{\partial^2 V_{eff}}{\partial\rho_i\partial\rho_j}\right) = 16a^4\left(1 + b - 2e + 3c - \frac{\tilde{\lambda}_3 v^2}{4a^2}\right).
$$

Thus the condition (2.16) is satisfied if

$$
b - 2e + 3c > -1 + \frac{\tilde{\lambda}_3 v^2}{4a^2}. \qquad (2.26)
$$

Now (2.23) is written as

$$
y^2(1-y)^2\frac{d^2\theta(y)}{dy^2} + y(1-y)(1-4y)\frac{d\theta(y)}{dy} = [b + c(1-y)^2 - e(1-y)]\sin\theta(y) + \frac{d}{2}(1-y)^2\sin(2\theta(y)). \qquad (2.27)
$$

This is the equation that we shall examine in detail. One sees that $\theta(y) = n\pi$ with $n \in \mathbf{Z}$ is the trivial solution. This equation is invariant under $\theta(y) \mapsto -\theta(y)$. This is because we have no explicit CP-violating terms in the potential.

Before closing this section, we comment on the ansatz adopted. Although the kinks (2.10) are solutions of (2.11) with the potential (2.18), they are no longer solutions of the coupled equations for ρ_i and θ_i with the potential (2.20). So that solutions to (2.23) will not be true solutions of the coupled equations. Further some of the parameters may be restricted to give a finite-energy solution for θ. We, however, expect that our solutions are not so different from the true solutions. This is because, as long as the ρ's have the kink shape, ρ's and θ's continuously rearrange themselves so as to take the minimal-energy configuration starting from our solutions. At the final stage, we should check that the physical quantities, such as the generated baryon number, are insensitive to small perturbations in the parameters of the potential.

3 Asymptotic Behaviors of θ

Before solving (2.27), we investigate the asymptotic behaviors of the solutions. This will help us to find numerical solutions. Among possible solutions, we are concerned in those with finite energy density. When all the gauge fields are gauged away, the classical energy of the bubble wall is given only by the contribution of the scalars:

$$E = \int d^3\mathbf{x} \left\{ \sum_{i=1,2} \left[\dot{\Phi}_i^\dagger(x)\dot{\Phi}_i(x) + \nabla\Phi_i^\dagger(x)\cdot\nabla\Phi_i(x) \right] + V_{eff}(\Phi_1, \Phi_2; T) \right\}.$$

For a static and planar bubble wall, the energy density per unit area is, in terms of ρ_i and θ_i,

$$\begin{aligned} \mathcal{E} &= \int_{-\infty}^{\infty} dz \left\{ \frac{1}{2}\sum_{i=1,2}\left[\left(\frac{d\rho_i}{dz}\right)^2 + \rho_i^2\left(\frac{d\theta_i}{dz}\right)^2 \right] + V_{eff}(\rho_1,\rho_2,\theta) \right\} \\ &= \int_0^1 dy \left\{ ay(1-y)\sum_{i=1,2}\left[\left(\frac{d\rho_i}{dy}\right)^2 + \rho_i^2\left(\frac{d\theta_i}{dy}\right)^2 \right] \right. \\ &\qquad \left. + \frac{1}{2ay(1-y)} V_{eff}(\rho_1,\rho_2,\theta) \right\}. \end{aligned} \tag{3.1}$$

In the case of the kink-type wall treated in the previous section, for the energy density to be finite, we must have

$$\theta_i'(y) \sim y^\alpha, \quad \text{with } \alpha > -1 \quad \text{for } y \sim 0, \tag{3.2}$$

$$\theta_i'(y) \sim (1-y)^\beta, \quad \text{with } \beta > -2 \quad \text{for } y \sim 1. \tag{3.3}$$

3.1 Asymptotic behavior in the broken phase

The potential with the kink-type profile (2.10) for ρ_i is written as

$$\begin{aligned} V_{eff}(\rho_1,\rho_2,\theta) &= 2a^2v^2y^2(1-y)^2 \\ &+4a^2v^2\sin^2\beta\cos^2\beta(1-y)^2\Big\{[b+c(1-y)^2-e(1-y)](1-\cos\theta) \\ &+\frac{d}{4}(1-y)^2(1-\cos 2\theta)\Big\}. \end{aligned} \tag{3.4}$$

Although in the symmetric phase the value of θ is not determined because of $V_{eff}(y=1)=0$, it will be allowed to take some specific values as shown later. On the other hand, in the broken phase ($y=0$),

$$V_{eff}(\theta_0) = 4a^2v^2\sin^2\beta\cos^2\beta\left[(b+c-e)(1-\cos\theta_0)+\frac{d}{4}(1-\cos 2\theta_0)\right], \tag{3.5}$$

where $\theta_0=\theta(0)$. If $d=0$ ($\lambda_5=0$), $\theta_0=2n\pi((2n+1)\pi)$ for positive (negative) $b+c-e$, *i.e.*, no CP violation. When $d\neq 0$, (3.5) is written as

$$V_{eff}(\theta_0) = -2a^2v^2d\sin^2\beta\cos^2\beta\left(\cos\theta_0+\frac{b+c-e}{d}\right)^2 + \theta_0\text{-indep. terms.} \tag{3.6}$$

This implies that

$$\begin{aligned} \cos\theta_0 &= \frac{b+c-e}{-d}, \qquad \text{for } d<0 \text{ and } |b+c-e|<-d, \\ \theta_0 &= 2n\pi, \qquad \text{for } 0<-d<b+c-e, \text{ or } d>0 \text{ and } b+c-e>0, \\ \theta_0 &= (2n+1)\pi, \qquad \text{for } b+c-e<d<0, \text{ or } d>0 \text{ and } b+c-e<0. \end{aligned}$$

Suppose that $\theta(y)$ is expanded as

$$\theta(y) = \theta_0 + y^\nu\sum_{n=0}^\infty a_n y^n \qquad (\nu>0, a_0\neq 0) \tag{3.7}$$

at $y\sim 0$. The constraint $\nu>0$ matches the condition (3.2). When inserted in (2.27), this yields

$$\begin{aligned} & y^\nu\Big\{\nu^2 a_0 + \Big[(\nu+1)^2a_1-\nu(2\nu+3)a_0\Big]y \\ & +\sum_{n=2}^\infty\Big[(n+\nu)^2a_n \\ & \quad -(n+\nu-1)(2n+2\nu+1)a_{n-1}+(n+\nu-2)(n+\nu+1)a_{n-2}\Big]y^n\Big\} \\ = \; & y^\nu\Big\{[b+c(1-y)^2-e(1-y)]\cos\theta_0\left[\sum_{n=0}^\infty a_ny^n - \frac{1}{3!}y^{2\nu}(\sum_{n=0}^\infty a_ny^n)^3+\cdots\right] \end{aligned} \tag{3.8}$$

$$
\begin{aligned}
&+\frac{d}{2}(1-y)^2\cos(2\theta_0)\left[2\sum_{n=0}^{\infty}a_ny^n-\frac{2^3}{3!}y^{2\nu}(\sum_{n=0}^{\infty}a_ny^n)^3+\cdots\right]\Bigg\} \\
+&\quad [b+c(1-y)^2-e(1-y)]\sin\theta_0\left[1-\frac{1}{2!}y^{2\nu}(\sum_{n=0}^{\infty}a_ny^n)^2+\cdots\right] \\
&+\frac{d}{2}(1-y)^2\sin(2\theta_0)\left[1-\frac{2^2}{2!}y^{2\nu}(\sum_{n=0}^{\infty}a_ny^n)^2+\cdots\right].
\end{aligned}
$$

In order to have a nontrivial solution, $2\nu \in \mathbf{Z}$.

When $\theta_0 = n\pi$ with $n \in \mathbf{Z}$, the lowest order terms give

$$(b+c-e)\cos\theta_0 + d = \nu^2. \tag{3.9}$$

The higher order terms of y will relate a_n with $n \geq 1$ to the lower coefficients in a nonlinear way.

When $\theta_0 \neq n\pi$, ν must be an integer for a nontrivial solution to exist. Then the lowest order terms yield

$$b + c - e + d\cos\theta_0 = 0. \tag{3.10}$$

For this to be satisfied,

$$|b+c-e| \leq |d|. \tag{3.11}$$

As we saw above, this θ_0 is energetically realized when $|b+c-e| < -d$ with $d < 0$. For $\nu = 1$, $O(y)$-terms of (3.8) lead to

$$a_0 = [(b+c-e)\cos\theta_0 + d\cos(2\theta_0)]a_0 - (2c-e)\sin\theta_0 - d\sin(2\theta_0),$$

which means, by use of (3.10),

$$a_0 = \frac{(2b-e)\sin\theta_0}{1+d\sin^2\theta_0}. \tag{3.12}$$

The higher order terms give relations among a_n with $n \geq 1$ and the lower coefficients. For $\nu \geq 2$, $O(y)$-terms of (3.8) yield $2b - e = 0$. When $\nu = 2$, $O(y^2)$-terms of (3.8) give

$$a_0 = -\frac{(b-e)\sin\theta_0}{4+d\sin^2\theta_0}. \tag{3.13}$$

On the other hand, $O(y^2)$-terms for $\nu \geq 3$ lead to $b = e$, which means $b = e = 0$. We discard this case because of two reasons: One is because nonvanishing $b \propto m_3^2$ is needed to violate CP spontaneously when $\lambda_{5,6,7} = 0$ as in the case of MSSM. The other is because m_3^2 in V_{eff} is induced in the presence of tree-level $\lambda_{5,6,7}$ so that to have $m_3^2 = 0$ is unnatural. Hence in this case, only $\nu = 1$ and $\nu = 2$ are allowed. Note that since $a_0 = \theta'(0)$, the initial conditions for (2.27) are completely fixed by the parameters of the potential.

To summarize, the possible boundary values of θ_0 are as follows:

(a) $\theta_0 = 2n\pi$, if $b+c-e > -d > 0$, or $d \geq 0$ and $b+c-e > 0$. The parameters must satisfy $b+c-e+d = \nu^2$ with $2\nu \in \mathbf{Z}$.

(b) $\theta_0 = (2n+1)\pi$, if $b+c-e < d < 0$, or $d \geq 0$ and $b+c-e < 0$. The parameters must satisfy $-(b+c-e)+d = \nu^2$ with $2\nu \in \mathbf{Z}$.

(c) $\cos\theta_0 = -(b+c-e)/d$, if $|b+c-e| < -d$ and $d < 0$, for $\nu = 1$ or 2.

Among these, we do not need to consider the case (b), as discussed in section 2.

3.2 Asymptotic behavior in the symmetric phase

Define $\zeta = 1-y$ and suppose that $\theta(\zeta)$ is expanded as

$$\theta(\zeta) = \theta_1 + \zeta^\mu \sum_{n=0}^{\infty} b_n \zeta^n \qquad (\mu > 0, b_0 \neq 0) \tag{3.14}$$

at $y \sim 1$ ($\zeta \sim 0$). In terms of ζ, (2.27) is written as

$$\zeta^2(1-\zeta)^2 \frac{d^2\theta(\zeta)}{d\zeta^2} + \zeta(1-\zeta)(3-4\zeta)\frac{d\theta(\zeta)}{d\zeta} = (b+c\zeta^2-e\zeta)\sin\theta(\zeta) + \frac{d}{2}\zeta^2\sin(2\theta(\zeta)). \tag{3.15}$$

The condition $2\mu \in \mathbf{Z}$ is required to have a nontrivial solution. The lowest order terms lead to

$$b\cos\theta_1 = \mu(\mu+2) \tag{3.16}$$

when $\theta_1 \equiv \theta(1) = n\pi$, and

$$b = 0 \tag{3.17}$$

and $\mu \in \mathbf{Z}$ when $\theta_1 \neq n\pi$. The higher order terms will give relations among the expansion coefficients. One may think that it is meaningless to ask the value of θ_1, since in the symmetric phase the Higgs fields vanish so that CP is never violated in the Higgs sector. But what is important for the baryogenesis is how the phases of them tend to some value as the Higgs fields disappear. We shall not discuss the case of $b = 0$, *i.e.*, $m_3^2 = 0$ for the reason stated above. Besides this, the first-order EWPT is realized when $b > -1$ (see (2.25)). Hence the possible boundary values θ_1 are;

$$\theta_1 = 2n\pi, \qquad b = \mu(\mu+2) \text{ with } 2\mu \in \mathbf{Z}. \tag{3.18}$$

4 Numerical Analysis

As we saw in the previous section, we are concerned with solutions satisfying the boundary conditions either (a) or (c), and (3.18), according to the parameters in the effective

potential. We present numerical solutions satisfying each set of boundary conditions. One corresponds to the case in which CP is spontaneously violated in the broken phase, while the other has no CP violation in the broken phase. To find such solutions, we performed numerical analysis using the relaxation, as well as the shooting, algorithms, with the parameters taken to satisfy the conditions (2.25) and (2.26). We also evaluated the chiral charge flux with use of these numerical solutions, which is the basic quantity to generate the baryon number in the charge transport scenario[2].

4.1 Solutions with spontaneous CP violation in the broken phase

Although one may think that nonzero θ_0 is induced in some model at finite temperature[3], we treat it as an input parameter. Once θ_0 is fixed, we can choose three of the four parameters (b, c, d, e) (see (c) in section 3). Because of (3.18), $b = 5/4, 3, 21/4, 8 \ldots$.

As an example, we show the solution for the case of $\theta_0 = 0.002$ and $(b, c, e) = (3, 7, 7)$ in Fig. 1. This suggests that the real part of the VEV has the kink shape and the imaginary part can be regarded as a perturbation to it. Then the effects of the CP violation on the fermions scattered off the wall, which interact with the Higgs through the Yukawa coupling, can be treated by the perturbative method developed in [4]. Recalling that $\rho_i \propto 1 - y$ for the kink-type profile, the behavior of $\theta(y) \sim \theta_0(1 - y)$ with small θ_0 in Fig. 1 implies that the imaginary part of the VEV is approximately proportional to the square of the real part, just as in the case studied in [5].[3] We shall not repeat the calculation of the chiral charge flux and note that $\Delta\theta$ in [5] should be replaced with θ_0 here.

Besides this solution, we also found a solution with $\theta_0 = 1$ for $(b, c, e) = (3, -1, 0)$, shown in Fig. 2. Unlike the above solution, it no longer has a linear shape and the perturbative method is not applicable. Such a solution would be realized when CP violation in the broken phase is enhanced by finite temperature effects and almost disappears at zero temperature as the case studied by Comelli, *et al.*[3].

4.2 Solutions without CP violation in the broken phase

In this case the parameters must satisfy (a). Then $(\rho_1, \rho_2, \theta) = (v \cos\beta(1 - y), v \sin\beta(1 - y), 2n\pi)$ is an exact solution to the full equations of motion (2.7) and (2.8), which we refer to as the trivial solution. If there exists a nontrivial solution to (2.27), it will open an interesting possibility that nonzero baryon asymmetry can be generated even in a CP-conserving theory in the broken phase.

[3] To $O(\theta_0)$, $\theta(y) = \theta_0(1 - y)^\mu$ with $2\mu \in \mathbf{Z}$ is an approximate solution to (2.27) for $b = \mu(\mu + 2)$, $e = \mu(2\mu + 5)$ and $d = \mu(\mu + 3) - c < 0$.

We found such a nontrivial solution for $(b, c, d, e) = (3, 12.2, -2, 12.2)$, whose profile is plotted in Fig. 3. From this, the real and imaginary parts of the VEV are obtained and shown in Fig. 4, which suggest that the perturbative method is applicable. We calculated the chiral charge flux for various choices of the fermion mass m_0 and wall width. The results are summarized as the contour plot in Fig. 5. This shows that the chiral charge flux is comparable to those studied in [5], so that it could generate sufficient baryon number for the thin wall case, taking into account the enhancement of forward scattering.

The energy density per unit area of this solution is

$$\Delta\mathcal{E} = \mathcal{E} - \mathcal{E}|_{\theta=0} = -2.056 \times 10^{-3}\, av^2 \sin^2\beta \cos^2\beta, \tag{4.1}$$

where $\mathcal{E}$ is defined by (3.1) and $\mathcal{E}|_{\theta=0} = av^2/3$. Since this solution is *not* a solution to the full equations of motion, the energy of the true solution may be lower than this value. We found that the solution is stable under perturbation of the parameters, so that we expect that this type of solutions will exist even if we do not impose the kink-type profile for ρ_i.

For such a nontrivial solution to exist, the CP-violating state must be favored in the intermediate region between the broken and symmetric phases. From (3.4), the effective potential along the kink is

$$\begin{aligned} V_{eff}(\zeta, \theta) \;=\; & 2a^2v^2 \Big\{ \zeta^2(1-\zeta)^2 \\ & -d\zeta^4 \sin^2\beta\cos^2\beta \left[\left(\cos\theta + \frac{b + c\zeta^2 - e\zeta}{d\zeta^2} \right)^2 - \left(1 + \frac{b + c\zeta^2 - e\zeta}{d\zeta^2} \right)^2 \right] \Big\}, \end{aligned} \tag{4.2}$$

where $\zeta = 1 - y$. Just as the boundary value θ_0, $\theta(y)$ can take a value other than $n\pi$, when $d < 0$ and there is a region in $0 < \zeta < 1$ such that $|(b + c\zeta^2 - e\zeta)/(d\zeta^2)| < 1$. Further minimization with respect to ζ in such a region will give the local minimum of V_{eff}. Since this new minimum appears on coupling the phase θ, the expected structure of $V_{eff}(\rho_1, \rho_2, \theta)$ is somehow modified compared with $V_{eff}(\rho_1, \rho_2, 0)$, which has degenerate local minima at $(\rho_1, \rho_2, \theta) = (0, 0, 0)$ and $(v\cos\beta, v\sin\beta, 0)$ reflecting the first-order nature of the EWPT. Hence we must require that the new minimum with $\theta \neq n\pi$ should not be so deep, otherwise it drastically changes the dynamics of the EWPT. Here we shall not investigate this condition further, but only assure that the new minimum is not lower than $(0, 0, 0)$. For the above set of the parameters and $\beta = \pi/4$, for which the negative contribution is maximal, V_{eff} as a function of (y, θ) is shown in Fig. 6. The new minimum of V_{eff} is about $0.0069a^2v^2$ and the height of the saddle point between it and the origin is about $0.038a^2v^2$, which is much lower than the maximum $0.125a^2v^2$ along $\theta = 0$. Since the barrier between the new minimum and the origin is not so high, we expect that the nature of the EWPT is not essentially altered. The contour plot of V_{eff} also shows that

the numerical solution for $\theta(y)$ goes around the maximum at $(1/2, 0)$. The value of V_{eff} where the maximum of $\theta(y)$ is reached is about $0.116a^2v^2$, which is just below the CP-conserving maximum. This fact suggests that this type of solution may exist irrespective of the depth of the new local minimum.

5 Discussions

Based on a rather general effective potential in the two-Higgs-doublet model at the transition temperature, which exhibits first-order EWPT, we have classified possible finite-energy bubble wall solutions with CP violation. We have found two types of numerical solutions, which are characterized by the CP-violating angle in the broken phase, θ_0.

One of them is the solution in the case where CP is spontaneously violated in the broken phase and will be the lowest-energy solution in that case. It smoothly mediates between the CP-violating vacuum in the broken phase and the CP-conserving one in the symmetric phase. For sufficiently small θ_0, the profile of the bubble wall is similar to that used in the literatures to estimate generated baryon numbers, while for $\theta_0 = O(1)$, $\theta(y)$ is no longer proportional to the kink so that the previous estimations of generated baryon number with large θ, by use of some presumed profile, should be revised.

The other connects the CP-conserving vacua in both phases. When CP is not violated in both vacua, there always exists the trivial solution with $\theta = 0$ all along. The new solution has nonvanishing θ within the bubble wall and has lower energy than the trivial one, as shown in (4.1). For the critical bubble of radius R_C, this CP-violating bubble will be nucleated with more probability than the trivial one by the factor

$$\exp\left(-\frac{4\pi R_C^2 \Delta\mathcal{E}}{T_C}\right). \tag{5.1}$$

According to the estimation in the massless two-Higgs-doublet model[6], the radius of the critical bubble is given by $\sqrt{3F_C/(4\pi a v^2)}$, where F_C is the free energy of the critical bubble and is found to be about $145T$. Then the exponent in (5.1) is $0.89 \sin^2\beta \cos^2\beta$, irrespective of the wall width. This solution will provide a new possibility to generate baryon number even within the framework of a CP-conserving theory.[4]

We estimated the generated baryon number with the use of each solution, based on the charge transport mechanism. With the enhancement of forward scattering, we could have sufficient baryon number for thin wall case. Although we did not work out on the spontaneous scenario, we expect that sufficient baryon number could be obtained if the effects of diffusion are considered[7].

[4] Strictly speaking, one needs a CP-odd term in the energy to have net baryon number by solving the degeneracy between the solutions with $\theta(y)$ and $-\theta(y)$.

We found that each type of the solutions are stable under perturbation of the parameters in V_{eff}, as long as they satisfy the required conditions. Hence these kinds of solutions will exist in wider class of models. It will be interesting to find a realistic model which admits such a solution.

When we almost completed this work, we noticed a work done by J. M. Cline *et al.*, in which they found solutions with $\theta_0 \neq 0$ ($|\theta_0| \ll 1$)[8]. They incorporated the case of the explicit CP breaking, besides the spontaneous case, and their equation for θ corresponds to that with $c = e = 0$ in our analysis.

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Figure Captions

Fig.1: The numerical solution of $\theta(y)$ for $\theta(0) = 0.002$ and $\theta(1) = 0$. The parameters are $(b, c, e) = (3, 7, 7)$.

Fig.2: The numerical solution of $\theta(y)$ for $\theta(0) = 1$ and $\theta(1) = 0$. The parameters are $(b, c, e) = (3, -1, 0)$.

Fig.3: The numerical solution of $\theta(y)$ for $\theta(0) = \theta(1) = 0$. The parameters are $(b, c, d, e) = (3, 12.2, -2, 12.2)$.

Fig.4: The profile of the bubble wall corresponding to the solution in Fig.3 with $x = az$. The solid line represents the kink, which is the absolute value of the VEV with the maximum being normalized to unity. The dashed line and dashed-dotted line are the real and imaginary parts of it, respectively.

Fig.5: Contour plot of the chiral charge flux, normalized as $F_Q/(uT^3(Q_L - Q_R))$, for $u = 0.1$ and $T = 100\mathrm{GeV}$, where u is the wall velocity and F_Q is defined in [5].

Fig.6: Contour plot of $V_{eff}/(a^2 v^2)$ along the kink, as a function of y and θ, for the parameters used to find the solution in Fig.3 and for $\beta = \pi/4$.

Fig.1

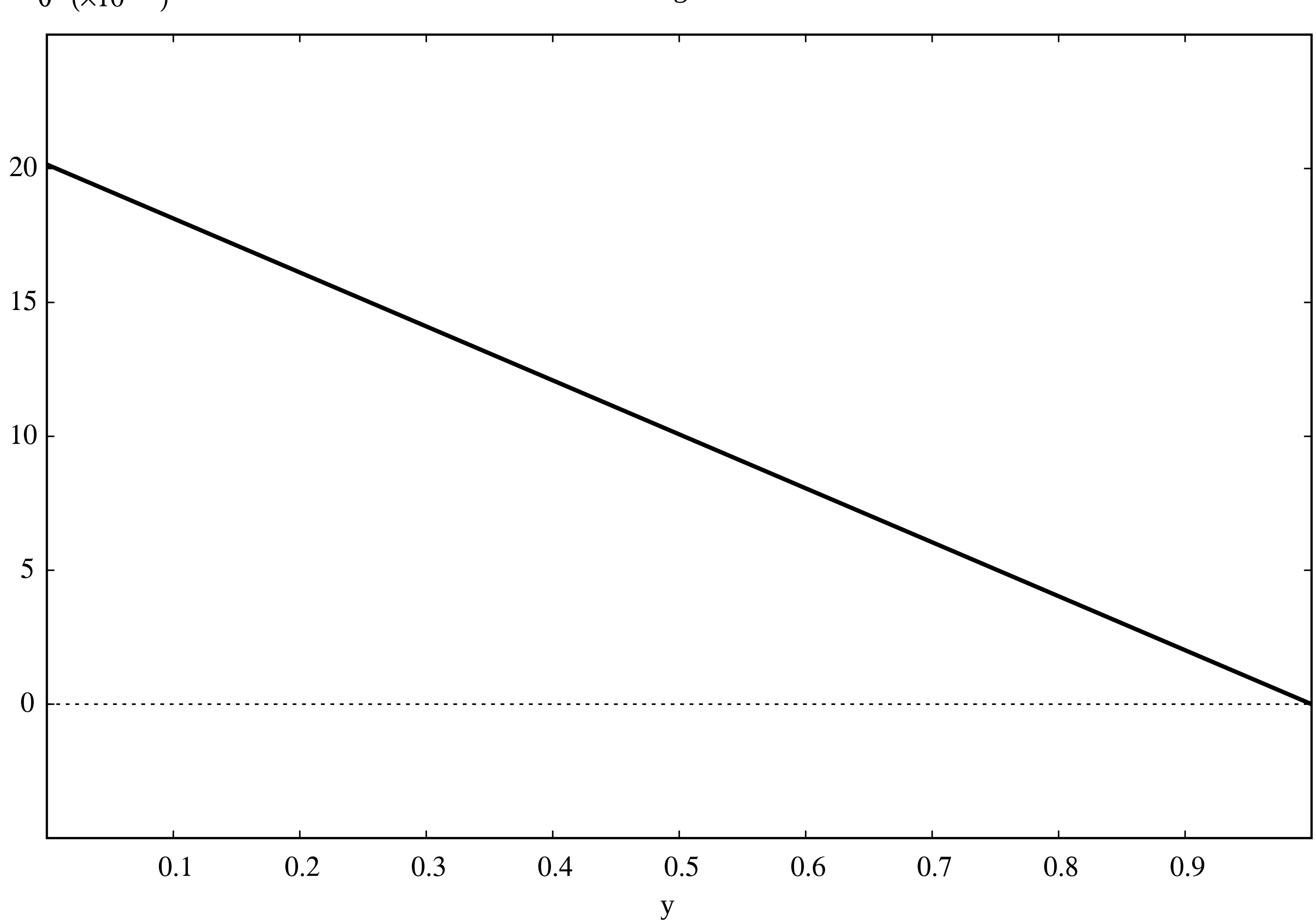


Fig.2

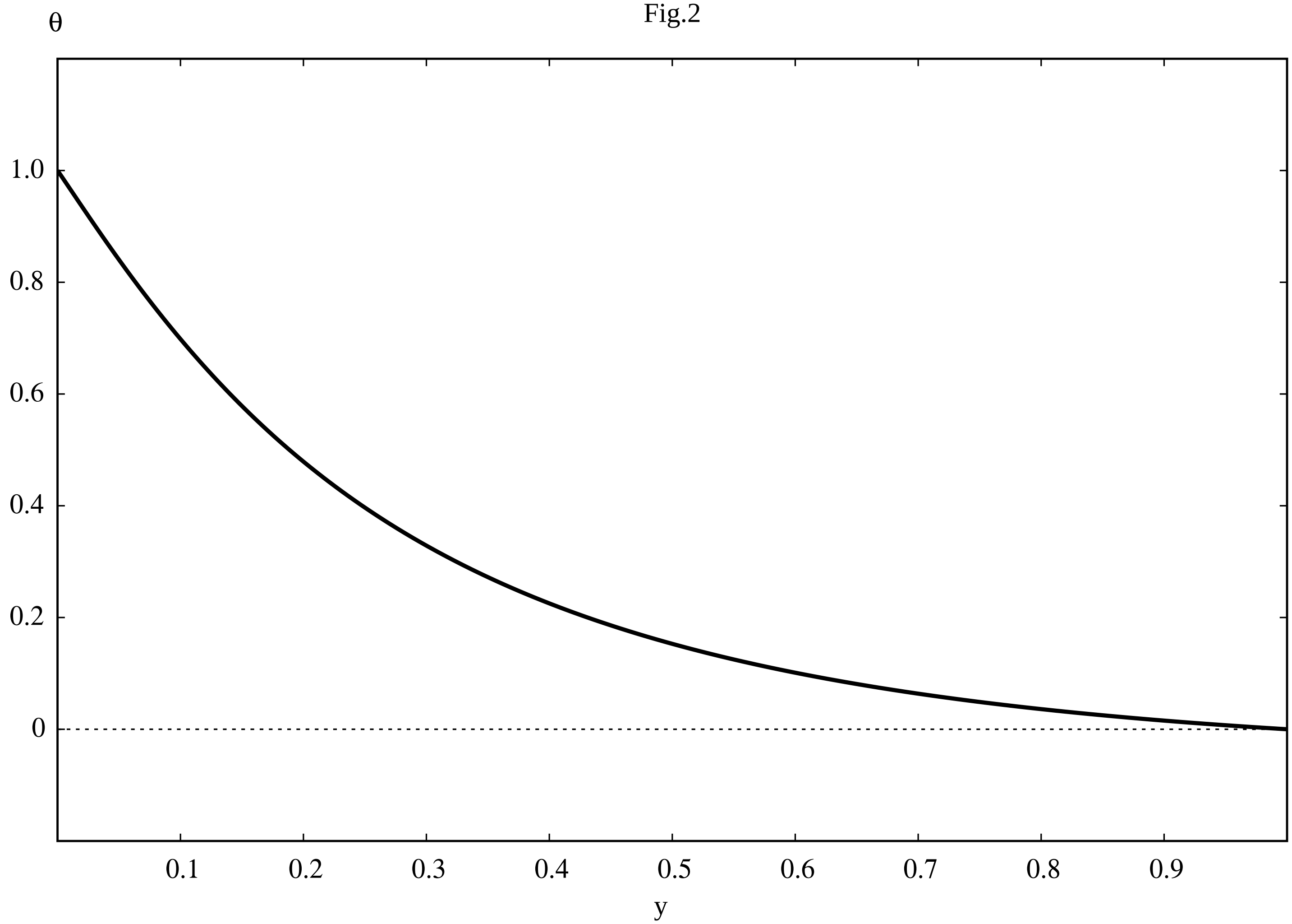


Fig.3

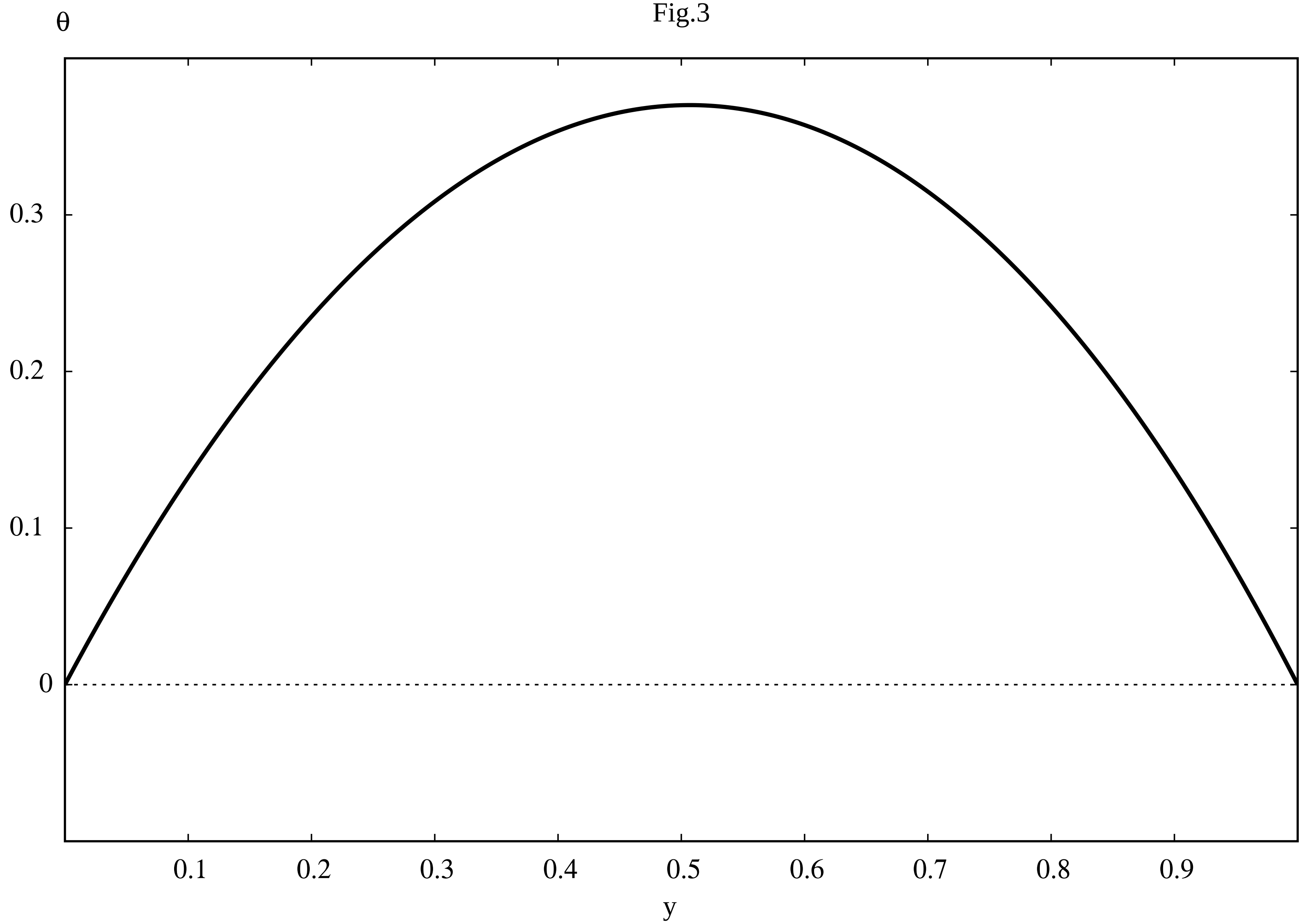


Fig.4

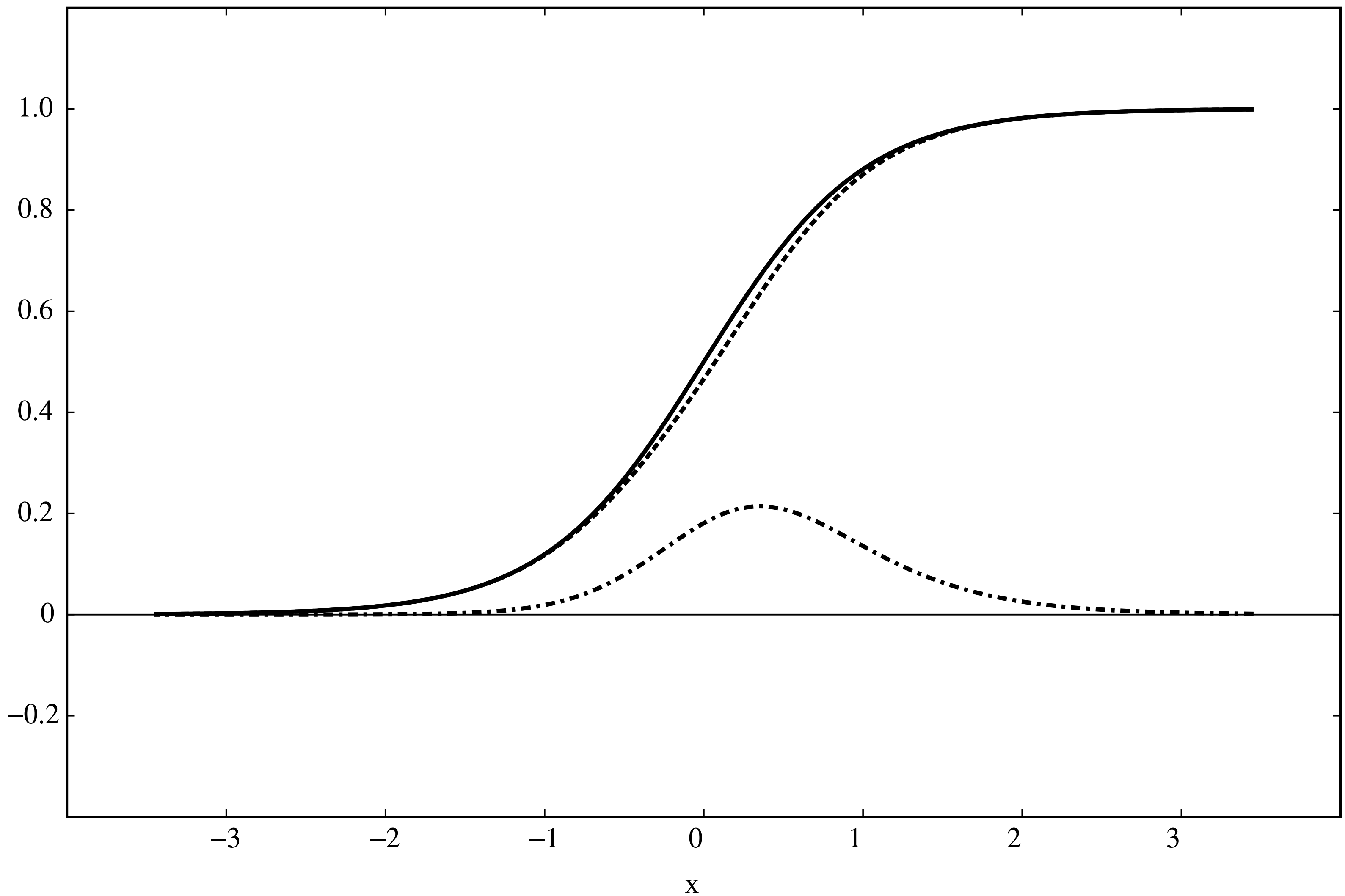


Fig.5

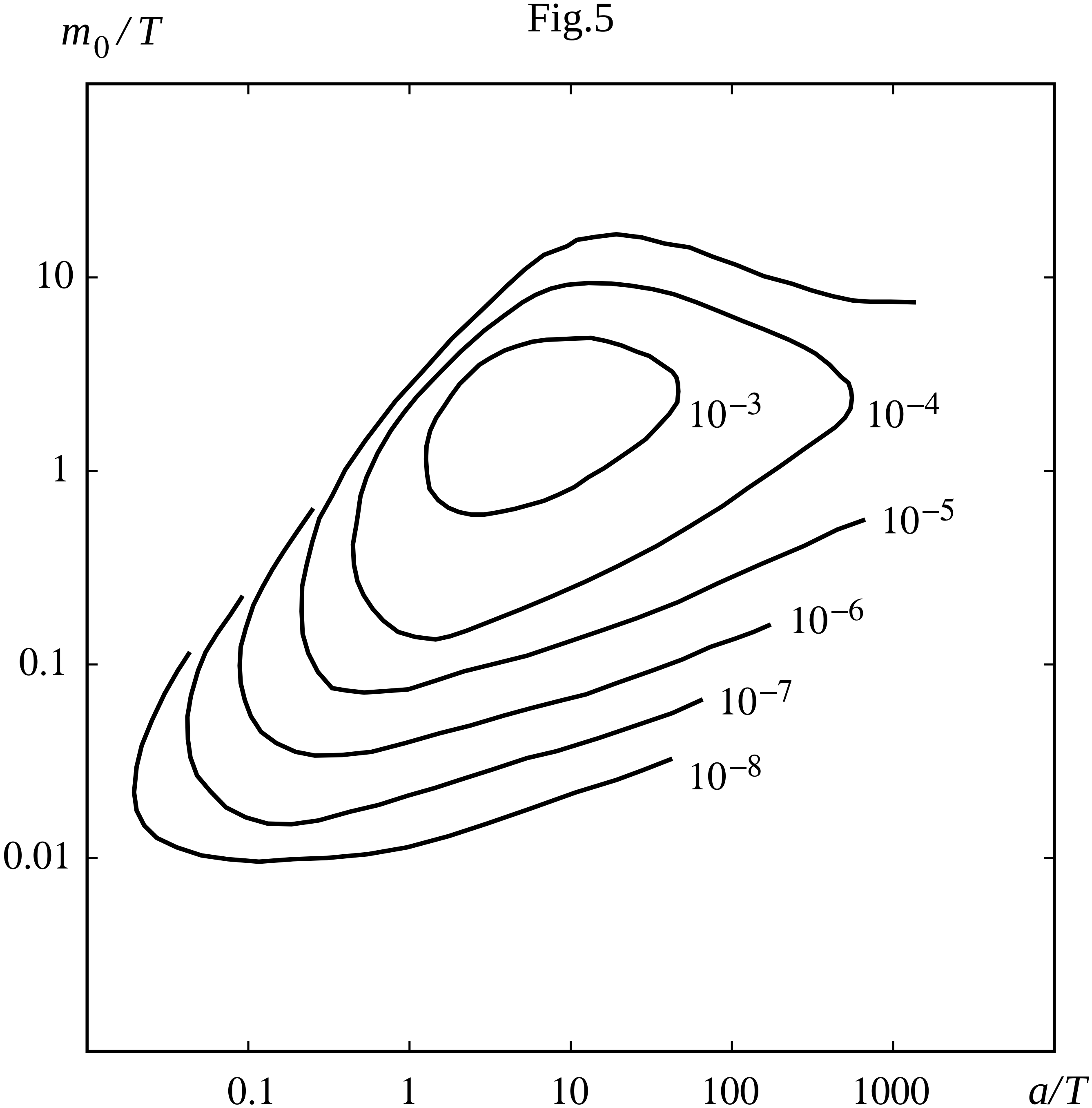
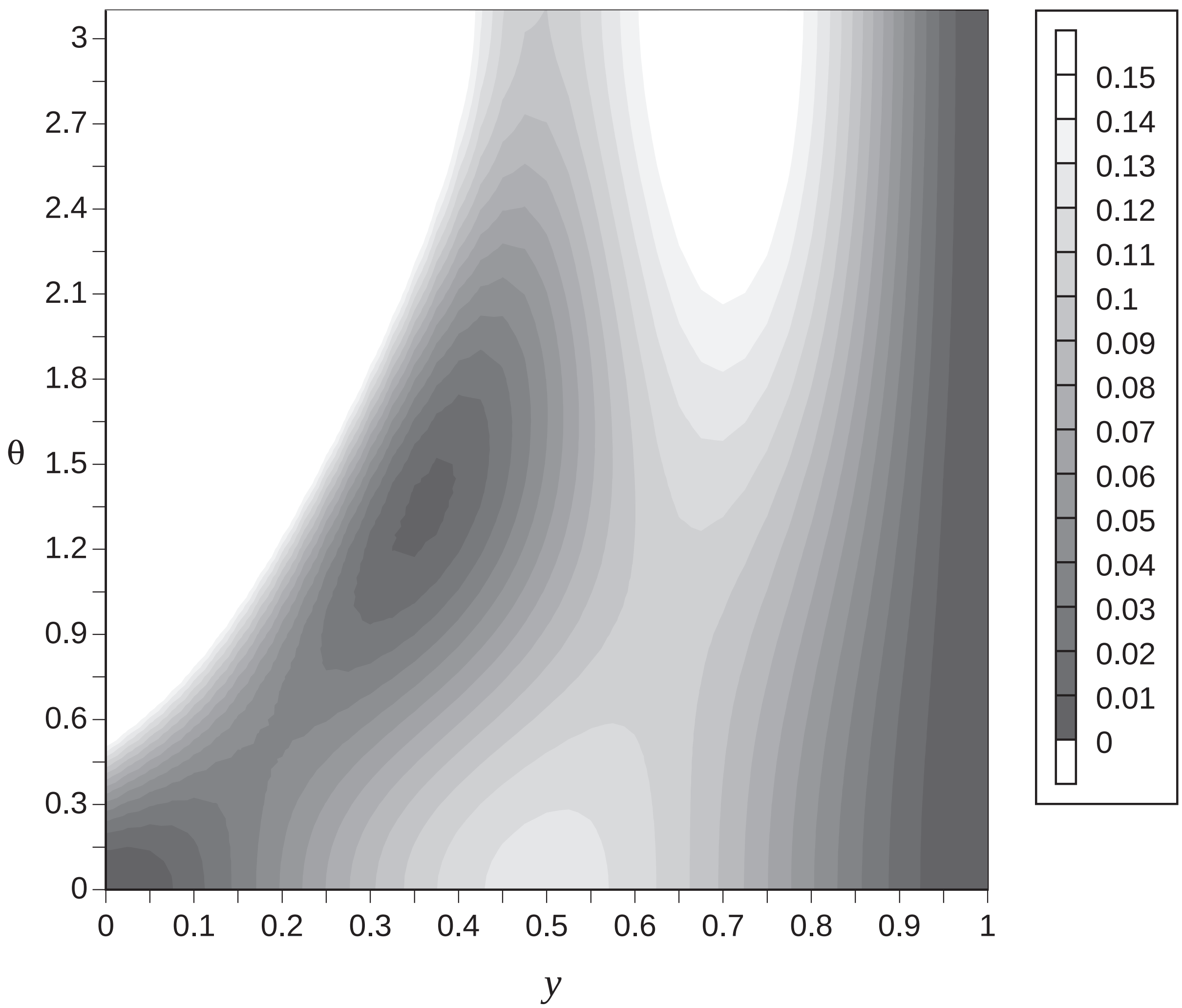


Fig.6



θ
3
2.7
2.4
2.1
1.8
1.5
1.2
0.9
0.6
0.3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
y
0.15
0.14
0.13
0.12
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0